Lab 5: DC Motors and the Step Response of First Order Systems

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1: Introduction

Direct Current motors (DC motors) utilize a direct current input and produce a rotational motion in their shaft, which is through a magnetic field that causes the rotor to rotate around the stator [1]. These are extremely common in everyday devices, such as simple pumps, fans, cars, or most devices with a rotational motion [2]. In Lab 5, a DC motor is used as a tachometer, which is a device used to measure the speed of a motor [1]. The response of the DC motor to an applied voltage can be viewed as a first order system. Through the first order differential, a time constant, which describes a system's response to a step input as a numerical value. The time constant determines how quickly a system reaches its steady state. These values will be analyzed through the use of three different methods - inspection, the central difference method, and linear regression.

A variety of parameters can be used to describe a tachometer and its results and functions. These include the time constant, calibration constant, and steady speed state. All of these are useful in finding unknown parameters of any given motor or tachometer. Theoretically, it is expected to see a similar calculated value for each parameter through each different method.

2: Methods

Prior to any data collection, the given values must be recorded. In this lab, the applied voltage was 12 volts, the motor winding resistance was 0.75 ohms, and the external resistor's resistance was 3.09 ohms. The measurements were taken via a tachometer. Due to the necessity for a conversion to RPM, the calibration constant is found with equation (1), using the steady state voltage and rotational speed to solve for c. The measurements for this lab were taken through the LabView Software. The motor was turned on and it was confirmed that it was working. For the motor-only data, the switch was turned to "MW Only", which allows the motor to run without the in-line resistor. Data was then collected for a short period of time as the motor was turned on, about 5 seconds. The process was repeated for data collection while including the in-line resistor. Using the steady state voltage output, the steady state speed of the motor, and equation (1) below, the voltage data sets were converted to RPM

$$\Omega = cV_s \tag{1}$$

The three methods were then used to calculate the parameters for this lab. By inspection, the 63.2% response point was found, and in turn the corresponding estimate of time constant. This is done by finding the point where rotational speed reaches 63.2% of its steady state value [3].

The next method used was the central difference method. This method consists of finding the derivative of a point on the steady state response curve to be used to create a tangent line to the curve. The intersection of this tangent line is then used to estimate the time constant. The derivative of the discretely sampled signal can be approximated with the central difference method, in equation (2), where h is the sampling period [3].

$$f'(x_j) = \frac{f(x_{j+1}) - f(x_{j-1})}{2h} \tag{2}$$

The derivative can then be used in equation (3) to find the slope of response at any given time. In this equation, Y is the steady state response and τ. A line tangent to the curve can be found using the slope at a given point.

$$\frac{dy}{dt} = \frac{Y}{\tau} e^{\frac{-t}{\tau}} \tag{3}$$

The last method for finding the time constant is using the linear regression method. This method tends to be more rigorous yet more accurate. The recorded data undergoes semi-log transformation, and then a linear regression is performed. It can be transformed into semi-log form by rearranging equation 3 above and finding the natural log of either side. With the new data, equation (4a) can be used. The slope of this equation is $-\frac{1}{\tau}$Therefore the time constant can be found with equation (4b) [3].

$$ln(1 - \frac{y(t)}{Y}) = -\frac{1}{\tau} t \tag{4a}$$

$$\tau = -1/(slope\ of\ regression) \tag{4b}$$

After finding the time constant using these three methods, other parameters must be found. Using a manufacturer supplied torque constant. The recorded steady state values and known system parameters can be used to determine the motor voltage constant and damping coefficient. The motor voltage constant can be calculated using the following equation (5). By using the time constants found with equation (4), a system of equations is created, and b and k_g can be solved for.

$$\tau = J/(b + \frac{K_t K_g}{R}) \tag{5}$$

Using these variables, the system's rotational inertia can be calculated. This can be done using the following equation (6).

$$\tau = J/(b + \frac{K_t K_g}{R}) \tag{6}$$

3: Results and Discussion

Figures 1 and 2 show the collected data from both the motor with no additional resistance, as well as the motor with the additional resistance.

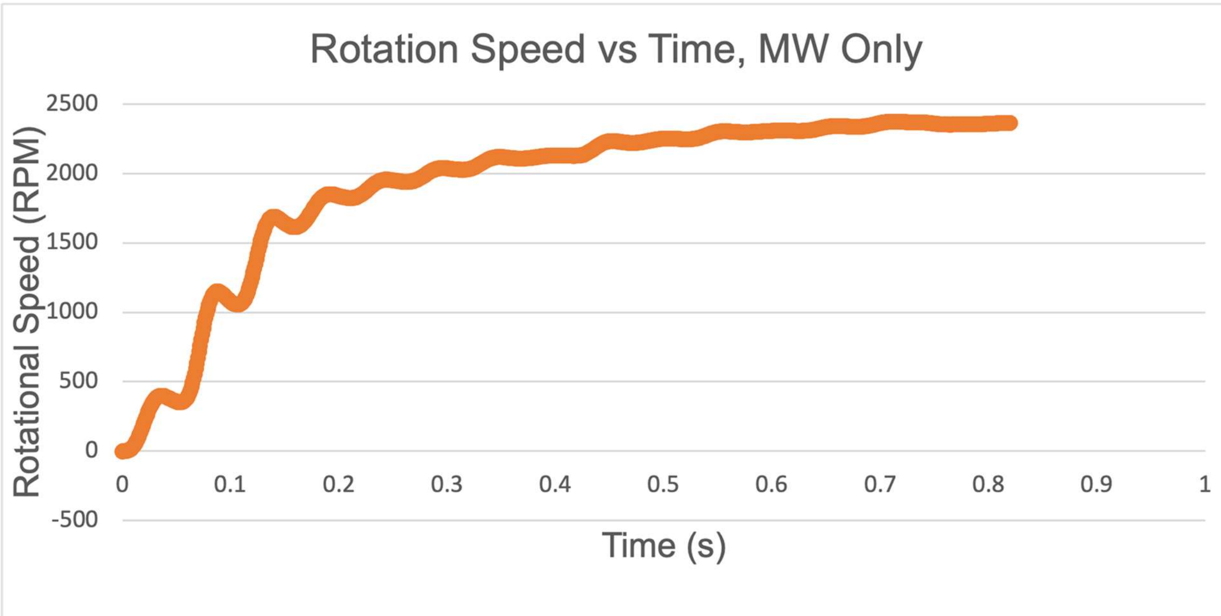

Figure 1: Graph for Rotation Speed vs Time without additional

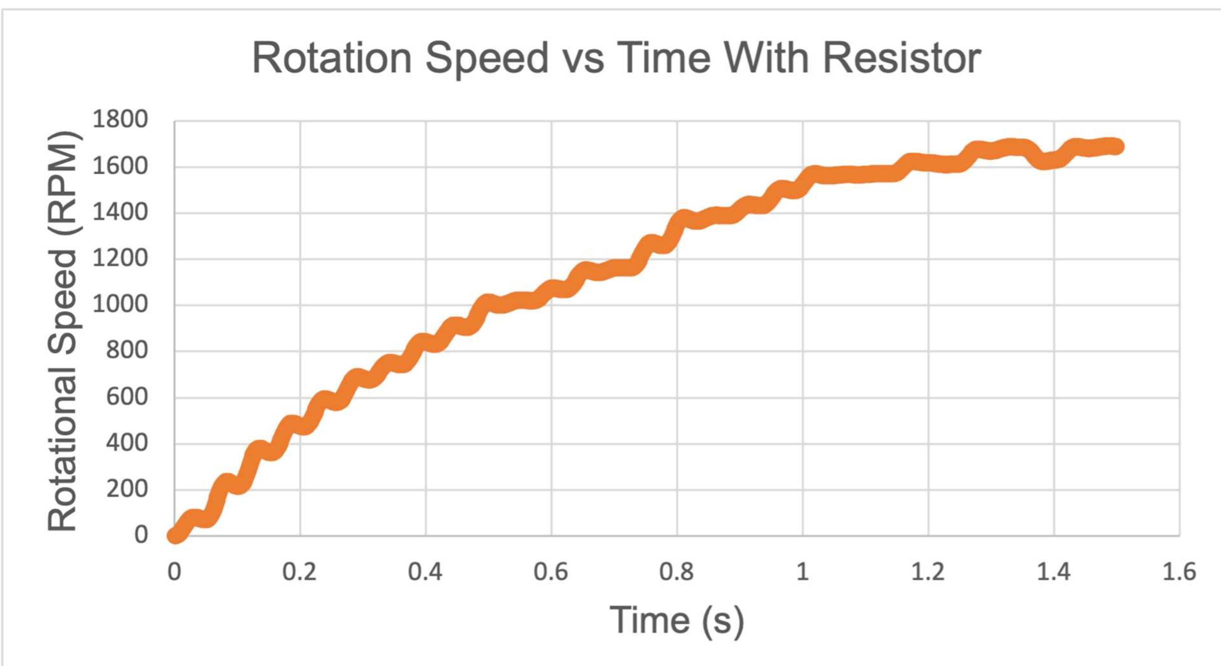

Figure 2: Graph for Rotation Speed vs Time with the additional resistor

Via inspection when rotational speed is approximately 63.2% of the steady state speed. This was found to be 0.13 seconds with just the MW, and 1.1 seconds with the additional resistor. Using the central difference method, a point was chosen at random, and the slope was calculated. The only MW motor time constant found was 0.18 seconds. For the motor with the resistance, the time constant found was of .43 seconds. The last method used was via the semi log transform method. The graphs of the data after undergoing a semi log transform as well as their lines of best fit are shown in figures 3 and 4.

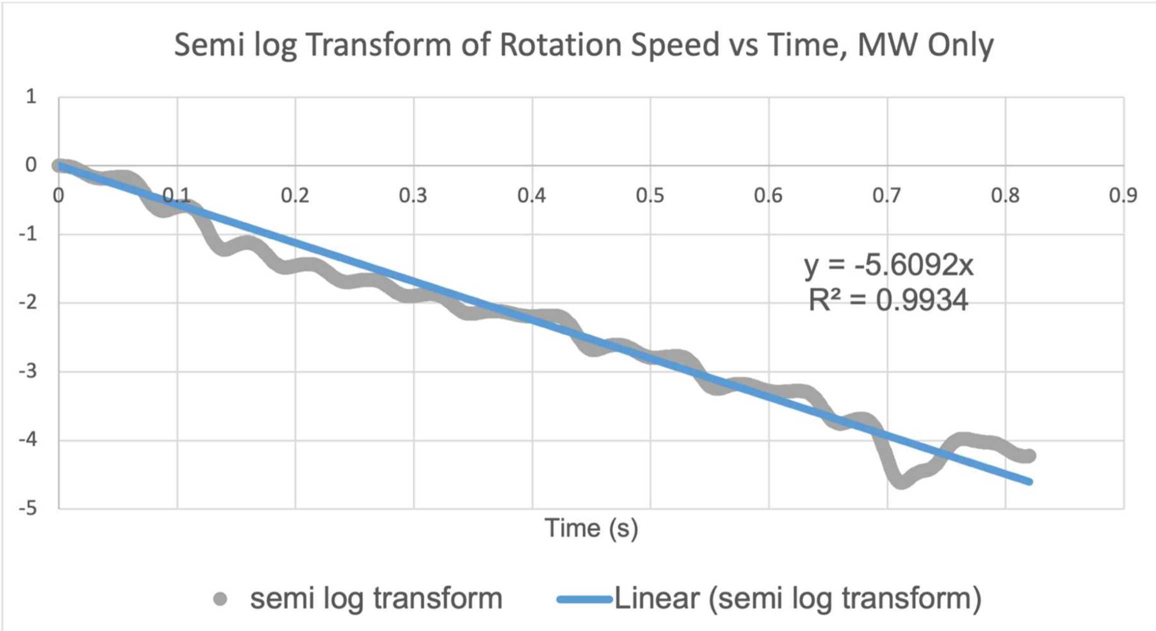

Figure 3: Graph for the semi log transform Rotation Speed vs Time without the

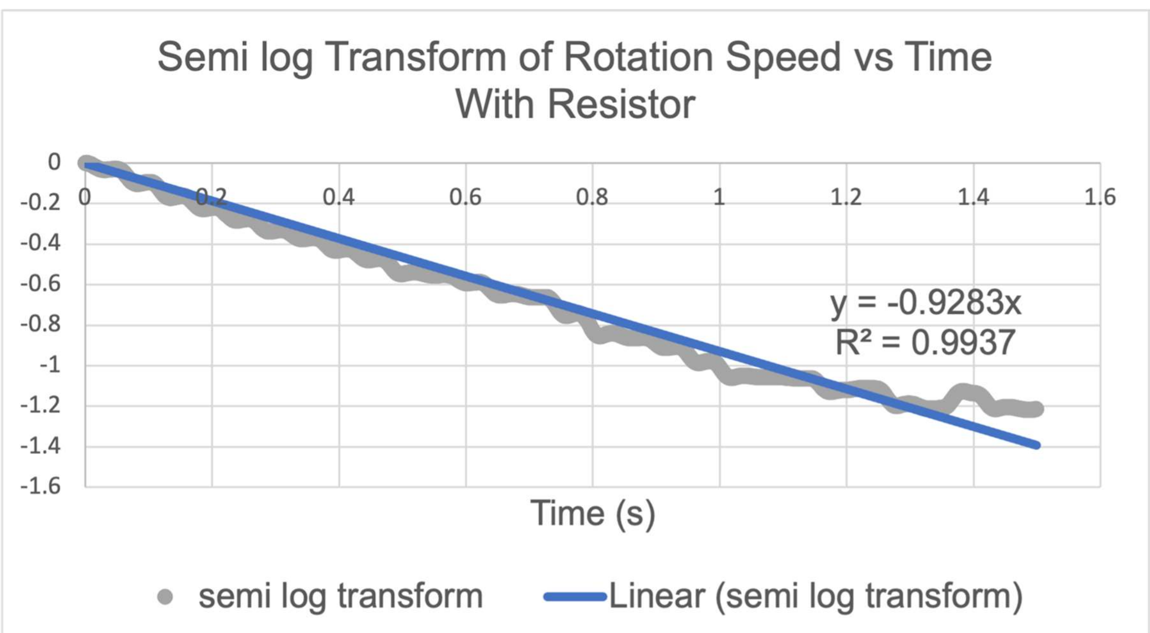

Figure 4: Graph for the semi log transform of Rotation Speed vs Time with the

Using equation (4b), the time constant for the MW only is .18 seconds. and the resistor is 1.08 seconds. These are likely the most accurate calculations for time constant. This implies that the method of inspection was capable of providing a good estimate as to where the time constant was. However, the central difference method was completely inaccurate.

Using equation (5), the following system of equations was created.

$$\frac{(2400RPM)}{12V} = 1/(K_g + \frac{b(0.75\Omega)}{(0.0461\ N{-}m/A)})$$

$$\frac{(1800RPM)}{12V} = 1/(K_g + \frac{b(3.84\Omega)}{(0.0461\ N{-}m/A)})$$

By solving this system of equations, $b = .0117\ Nm/RPM$ and $K_g = 0.00459\ V/RPM$. Using these values, the time constants found via linear regression, and equation (6), the rotational inertia of the MW only motor is $J_{MW} = 37.62\ kgm^2$, and the resistor winding is $J_{resistor} = 44.44\ kgm^2$.

4: Conclusion

The three methods of determining a time constant have varying levels of reliability. The method of inspection seems reliable and is consistent. This method is so simple that it can be used to easily determine the general area of the correct value. The central difference method is extremely unreliable. The noise in the data created an oscillating effect instead of a perfect exponential curve. This means that the slope found at every point has very high variance, with some even being negative. Since there is no way to pick a point to check that will be accurate to the entire system, it is inconsistent and unreliable. If the data had controlled for this error more successfully, this method would be more reliable. Lastly, the most accurate measure was the linear regression method. By taking the line of best fit, the random error in the data was accounted for. The high correlation coefficient also implies that the slope found is accurate. The rotational inertia should not change regardless of the resistance within the motor. Since the inertia found with the MW time constant is within 15% of the inertia found with the resistance, the data collected, including the calculated time constants, were accurate. This shows that using a DC motor as a tachometer is acceptable. The difference can be attributed to the compounded error from the collected data. Moreover, the given values were not found experimentally, but given by the manufacturer. This data could have high variance, and the values like steady state rotational speed could have changed due to the age of the motor.

The data collection shows some oscillating motion in the rotational speed. The type of motion implies that the motor wasn't spinning at a constant speed, which could be attributed to its condition. In addition, random ambient forces in the room could have an impact on rotational speed. To correct for this in the future, a new motor could be used. Furthermore, multiple trials can be taken to verify the time constants found, reducing any random error. When finding the time constant via the central difference method, which point chosen has a large effect on the outcome. To control for this, multiple points should be tested. In addition, having data with a larger step size and less variation will improve the accuracy of this method.

5: References

[1] M. Huber, "DC Motors and the Step Response of First Order Systems" ME 3263 - Mechanical Engineering Lab, UConn School of Engineering, Fall 2023. [Online]. Accessed on November 15, 2023.

[2] Parvalux, “Everyday items that use DC Motors,” Parvalux, https://www.parvalux.com/us/4-everyday-items-that-use-dc-motors/ [Online]. Accessed on November 10, 2023.

[3] M. Huber (2023). ME 3263 Lecture 19: “DC MOTORS, TACHOMETERS, CALIBRATION, LAB 5”. [Online]. Accessed on November 15, 2023.